3[rd] series

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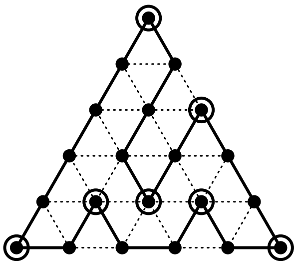
Address: *Korespondenční seminář iKS*
KAM MFF UK
Malostranské náměstí 25
118 00 Praha 1
Czech republic

Problem C3. An equilateral triangle with sides of length n is filled with a triangular grid. A closed path travels along the grid, visiting each vertex of the grid exactly once. Prove that this path turns in an acute angle at least $n+1$ times.



Problem G3. In a tetrahedron $ABCD$, the sum of areas of its faces ABC and ABD is equal to the sum of areas of the faces CDA and CDB. Show that the midpoints of the edges AC, AD, BC, BD and the incenter of $ABCD$ lie in a single plane.

Problem A3. Given real numbers $x_1, x_2, \dots, x_n$, show that for any non-empty subset $M \subset \{1, 2, 3, \dots, n\}$, the following inequality holds:

$$\Big(\sum_{i \in M} x_i\Big)^2 \leq \sum_{1 \leq i \leq j \leq n} (x_i + \cdots + x_j)^2.$$

Problem N3. Find all positive integers n for which the sets of prime divisors of n and $2^n + 1$ are identical.