1st series

Date: 4^{th} May 2015

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Problem N1. Does there exist an infinite progression of positive integers $a_1, a_2, ...$ such that a_i and a_j are coprime if and only if $|i - j| = 1$?

Problem A1. Patrik found n positive real numbers $a_1, a_2, \ldots, a_n$. Show that he is able to choose n numbers $x_1, x_2, \ldots, x_n$ from the set $\{-1, 1\}$ in such a way that the inequality

$$\sum_{i=1}^{n} x_i a_i^2 \geq \left(\sum_{i=1}^{n} x_i a_i \right)^2$$

holds.

Problem C1. A square is cut into triangles in such a way that no two vertices of these triangles are collinear. For each vertex (including the original vertices of the square), we count the edges (sides of triangles) touching it. Can all those numbers be even?

Problem G1. In a non-equilateral triangle ABC, let's denote the circumcenter, incenter and incircle by O, I and k, respectively. Let t be the tangent of k parallel to BC that doesn't contain BC. Let D and R be points on t such that D lies on OI and RI is perpendicular to OI. Show that the quadrilateral $RADO$ is cyclic.