4rd series

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Problem C4. Patrik drew a rectangular grid and marked a path on it. The path starts in the top left corner, ends in the bottom right one and passes through each vertex exactly once. Then, Patrik coloured the areas (bounded by the marked path) that are adjacent to the left or bottom border of the grid. Prove that there are equally many coloured and non-coloured grid cells.

Problem G4. In a triangle ABC, the angle bisector at vertex A intersects side BC at point D. Let's denote the midpoint of segment AD by M. Segments MB, MC intersect circles with diameters AC, AB again at points X, Y, respectively. Prove that points D, M, X, Y lie on one circle.

Problem N4. There's an odd prime p and a sequence that satisfies $a_n = a_{n-1} + a_{n-p}$ for $n \geq p$ and $a_n = n$ for $n = 0, \ldots, p-1$. How many numbers out of $a_0, a_1, \ldots, a_{p^3}$ are divisible by p?

Problem A4. Find all functions that satisfy the equation

$$f(f(x) - f(y)) = f(f(x)) - 2x^2 f(y) + f(y^2)$$

for all $x, y \in \mathbb{R}$.