5th series

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Address: *KMS – iKS*
OATČ KAGDM FMFI UK
Mlynská dolina
842 48 Bratislava
Slovakia

Problem G5. Patrik has just bought a fancy *3D* television. He expects it to have proper *3D* content, too!
A tetrahedron $ABCD$ is given. A sphere s passing through point A intersects segments AB, AC, AD in points E, F, G, respectively. The intersection of sphere s and the circumsphere of tetrahedron $ABCD$ is a circle lying in a plane parallel to the plane BCD. Let us denote the reflections of points E, F, G with respect to the midpoints of segments AB, AC, AD by R, S, T, respectively. Prove that points B, C, D, R, S, T lie on one sphere.

Problem N5. There are n $(n \geq 2)$ pairwise coprime positive integers. The remainder of the product of arbitrary $n-1$ of them when divided by the remaining integer is always r. Prove that $r \leq n-2$.

Problem C5. There are two sufficiently long strips of paper. The letter A is written on the first strip and the letter B is written on the second one. In each move, we choose one strip and copy all the letters written on it in the same order on the other strip, either in front or at the end of its content. Prove that, after 2014 moves, it will be possible to split the first strip into two palindromes.

Problem A5. There's a positive real number written in each cell of an $n \times n$ table, such that the sum of numbers written in each row is equal to 1. Whenever we choose n cells such that exactly one cell is chosen from each row and each column, the product of their numbers is less than or equal to the product of the diagonal numbers. Prove that the sum of the diagonal numbers is at least 1.