3rd series

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Problem G3. In a triangle ABC, we chose points $A_1,\ B_1,\ C_1$ on sides $BC,\ CA,\ AB$ respectively, in such a way that $|AB_1| - |AC_1| = |CA_1| - |CB_1| = |BC_1| - |BA_1|$ holds. Let $I_a,\ I_b,\ I_c$ be the incenters of triangles $AB_1C_1,\ A_1BC_1,\ A_1B_1C$ respectively. Show that the circumcenter of triangle $I_aI_bI_c$ coincides with the incenter of triangle ABC.

Problem C3. Is it possible to assign a non-zero real number to each point of a plane in a way that makes the sum of numbers in the vertices of every regular 2015-gon equal to zero?

Problem A3. Patrik wrote a real monic polynomial of degree 2014, evaluated it in 2015 distinct integer points, considered absolute values of those numbers and took the largest one. He chose the polynomial and points that it was evaluated in, making sure that the number he took was the minimum possible. What was that number?

Problem N3. Patrik loves primes. Some of them he loves more, some he loves less, but he has some primes which he likes the most. He's hidden these primes in a finite non-empty set P. He'd like his birthday present to be an integer n, which can be written as $a^p + b^p$ for some $a, b \in \mathbb{N}$ (p is a prime) if and only if $p \in P$. Decide whether it's possible to fulfill his wish for any set P.